There is one more important concept in Modulus that I want to discuss. Once it is done, we will bid farewell to Mods (for the time being at least), I promise. The concept involves dealing with multiple Mod terms (I will explain in just a minute). Before I start with the discussion, let me point out that it is relevant only if you are looking for a 50/51 in Quant and if you are looking for a 50/51 in Quant, then it is definitely relevant (I remember seeing a mean Modulus question in my GMAT a while back). But remember, don't waste time on advanced Modulus questions if you are uncomfortable with Number Properties or other such high-weightage topics. Only when you are above 48 consistently in Quant, should you spend time on the two posts titled 'Holistic Approach to Mods'. That said, everyone is welcome to read the posts and elicit his/her own takeaways.

Let's start.

Our problem for today is:

For what value of x, is $|x – 3| + |x + 1| + |x| = 10$?

The one good thing about GMAT is that it gives you five options. Here, the five options will be the possible values of x. So obviously, we will not waste time solving this question. We will just plug in the values and find out which value gives the sum 10. So let me write the complete question here:

For what value of x, is $|x – 3| + |x + 1| + |x| = 10$?

(A) 0

(B) 3

(C) -3

(D) 4

(E) -2

When you put x = 4, you get $|4 – 3| + |4 + 1| + |4| = 10$. So answer is (D).

When I made this question and was putting the options, obviously I had to solve the question to find the value of x (one option has to be the correct answer after all!) Using the method I will just discuss, I did it in my mind in under a minute! Curiosity piqued? I hope so.

Let us change the question a little to rope you in.

For how many values of x, is $|x – 3| + |x + 1| + |x| = 10$?

(A) 0

(B) 1

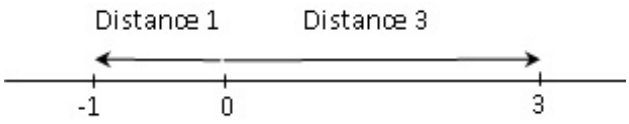
(C) 2

(D) 3

(E) Infinite

(Previously we discussed that Modulus represents distance. In this post, we will build upon that concept.)

In simple language, the question tells us that x is a point on the number line such that the sum of its distance from 3, -1 and 0 is 10. Let's say, if x = 0,



Distance of 0 from 3 = 3

Distance of 0 from -1 = 1

Distance of 0 from 0 = 0

Sum of distance of 0 from 3, -1 and 0 is 3 + 1 + 0 = 4.

So we know that x is not equal to 0. Now let's see what happens if x = 3.

Sum of distances of 3 from 3, -1 and 0 is 0 + 4 + 3 = 7. We need 3 more units of distance to make it 10. We need to make x go a little more to the right. Tell me, what happens when x goes 1 unit to the right? By how much will the distance increase? By 3 units! Because x will be 1 unit away from each of the 3 points. When x = 4, sum of distances of 4 from 3, -1 and 0 is 1 + 5 + 4 = 10.

So 4 is definitely one solution for x. What if we go further to the right? Every one unit further to right increases the distance by 3 units. So distance will keep increasing and will never be 10 again on this side of the number line.

Let's go to the other side. What happens if x = -1? Sum of distances of -1 from 3, -1 and 0 is 4 + 0 + 1 = 5. To increase the distance, we need to go further to left. Remember, the same logic holds here – Every one step to left will increase the distance by 3 units. We need to increase the distance by 5 units. So we take 1 step to the left (reach -2) and then take 2/3rd of a step to the left (reach – 2.667). So x = -2.667 is another solution. Now, every time we take another step to the left, the total distance will increase.

Therefore, there are only two solutions for x: 4 and -2.667

To review:

$$|x-3|+|x+1|+|x|=10$$

The Red line shows the region where the total distance of x from the 3 points is less than 10. The Blue lines show the region where the total distance of x from the 3 points is more than 10. The points -2.667 and 4 are the points where the total distance of x from the 3 points is 10.

Since there are 2 points where the total distance is 10, answer is (C).

A few things to ponder upon:

- I change the question to ‘For how many values of x, is $|x – 3| + |x + 1| + |x| = 4$?’ The answer now is 1. Why?

- I now change the question to ‘For how many values of x, is $|x – 3| + 3|x + 1| + |x| = 4$?’ The answer now is 0. Why?

- What happens if I change the question to ‘For how many values of x, is $|2x – 3| + |x + 1| = 10$?’

- What if I change it to ‘For how many values of x, is $|x – 3| – |x + 1| + |3x| = 10$?’

Thoughts on the points above are welcome. By the way, if a doubt arises somewhere, feel free to let me know and I will get back to you.